PORTFOLIO MANAGEMENT

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What is a portfolio?

In finance, portfolio is a set of assets or investments of an individual. An asset is for example stock, bond, cash, house you own or any other valuable property. Investors buy assets to earn the return, but any asset value might change and this change is representing the risk.

Investors prefer high return and low risk. But higher return is always linked to higher risk. How to decrease the risk and keep the high return? One option is to buy different assets and hope, that the decrease in value of one asset will be compensated by the increase in value of the other asset. This is the reason why investors build a portfolio. Why a good portfolio decreases the risk?

What does an investor prefer?

Let us first explain what investors prefer with respect of return and risk.

Return.

Investors prefer high return. Expected rate of return of the investment is calculated by following formula:

$$\text{Expected rate of return} = \frac{CF + P1 - P0}{P0}$$

Expected rate of return depends on cash inflow of the investment during the investment period (CF) and the capital gains or losses which depend on the change in value of the investment (price in period 1 (P1) minus price at the beginning (P0)).

Example:

If you expect to buy an Apple share in 1. January 2021 for 200 US dollar, you expect to sell it in 31. December 2021 for 220 US dollar and over the year 2021 you expect to receive dividend of 4 US dollars, your total expected rate of return is as follows:

$$\text{Expected rate of return} = \frac{4 + 220 - 200}{200} = 0{,}12$$

Your expected return is 12%. The expected return is often in practice calculated based on historical returns of the investment as Average rate of return and not only on expectations of the investor. For Apple inc. let us show an example based on historical data 2009-2018:

Date	Closing Price in USD	Dividend in USD	Annual return (Rn)
31.12.2008	12.88	-	
31.12.2009	27.44	-	113.09%
31.12.2010	48.47	-	76.67%
31.12.2011	65.21	-	35.71%
31.12.2012	65.07	0.58	1.57%
31.12.2013	71.51	1.17	12.74%
31.12.2014	117.16	1.85	66.67%
31.12.2015	97.34	2.03	-15.01%
31.12.2016	121.35	2.23	27.19%
31.12.2017	167.43	2.46	40.30%
31.12.2018	157.74	2.82	2.66%
Average rate of return, (Re Apple)			36.16%

The average rate of return is calculated as the average annual return based on individual years 2009-2018. We can conclude, that over the period 2009-2018 investor earned the average annual return 36.16% by holding Apple inc.

Risk

Risk of individual investment is based on the formula of standard deviation. To calculate the risk we use standard deviation, the symbol for which is σ , pronounced "sigma." The smaller the standard deviation, the lower the riskiness of the investment.

The formula for the standard deviation is as following:

$$\sigma = \sqrt[2]{\sum_{n=1}^{N}(R_n - R_e)^2 * P_n}$$

In our case the risk of Apple would be calculated as follows:

$$\sigma = \sqrt[2]{\sum_{n=1}^{N}(113,09 - 36,16)^2 * 0,1 + (76,67 - 36,16)^2 * 0,1 + \cdots + (40,3 - 36,16)^2 * 0,1 + (2,66 - 36,16)^2 * 0,1}$$

$$\sigma = 37,52\%$$

So we can conclude, that the investor by buying Apple shares can expect a return of 36.16% with risk 37.52%.

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What if the investor will buy not only Apple inc. but also Google and would build a portfolio? How would this effect the return and risk of the investor?

Let us calculate the return and risk also for Google inc:

Date	Closing Price in USD	Dividend in USD	Annual return (Rn)
31.12.2008	168.6329		
31.12.2009	263.9805	-	56.54%
31.12.2010	299.0591	-	13.29%
31.12.2011	288.9719	-	-3.37%
31.12.2012	376.434	-	30.27%
31.12.2013	588.28	-	56.28%
31.12.2014	533.0565	-	-9.39%
31.12.2015	742.95	-	39.38%
31.12.2016	796.79	-	7.25%
31.12.2017	1169.94	-	46.83%
31.12.2018	1035.61	-	-11.48%
Average rate of return, (Re Google)			22.56%
Standard deviation of return			25.29%

A portfolio

A portfolio is a set of assets or investments of an individual. Let us explain the effect of combining asset in a portfolio on expected return and risk.

The expected return is calculated as the average expected return on assets in portfolio:

$$R(p) = \sum_{n=1}^{N} R_n * w_n$$

R(p)   expected portfolio return

R(n)   expected individual asset return

w(n)   weight of asset in portfolio

If for example you combine two assets (Apple and Google) in a portfolio, the expected portfolio return will depend on average expected return of these two assets and the amount of money (weight) you invest into each asset (Apple and Google) expressed in %.

In our case, if you invest 40% of your money into Apple and 60% into Google your expected return based on previous calculations is:

$$R(p) = 36{,}16 * 0{,}4 + 22{,}56 * 0{,}6 = 28{,}00\%$$

The portfolio risk depends on a set of variables:

Risk of assets in the portfolio

Weight, how much money you invest in each asset included in the portfolio in %

Correlation of assets. Correlation describes the relation of return of individual assets. It ranges from -1 to 1.

If correlation is -1 it means that returns of asset A grow and the same time the returns of asset B go down with the same power. We say that returns are in this case negatively correlated.

If correlation is 1 it means that returns of asset A grow and the same time the returns of asset B grow also with the same power. We say that returns are in this case positively correlated.

If correlation is 0 it means that if returns of asset A grow returns of asset B sometimes go down and sometimes grow. We say that returns are in this case unrelated.
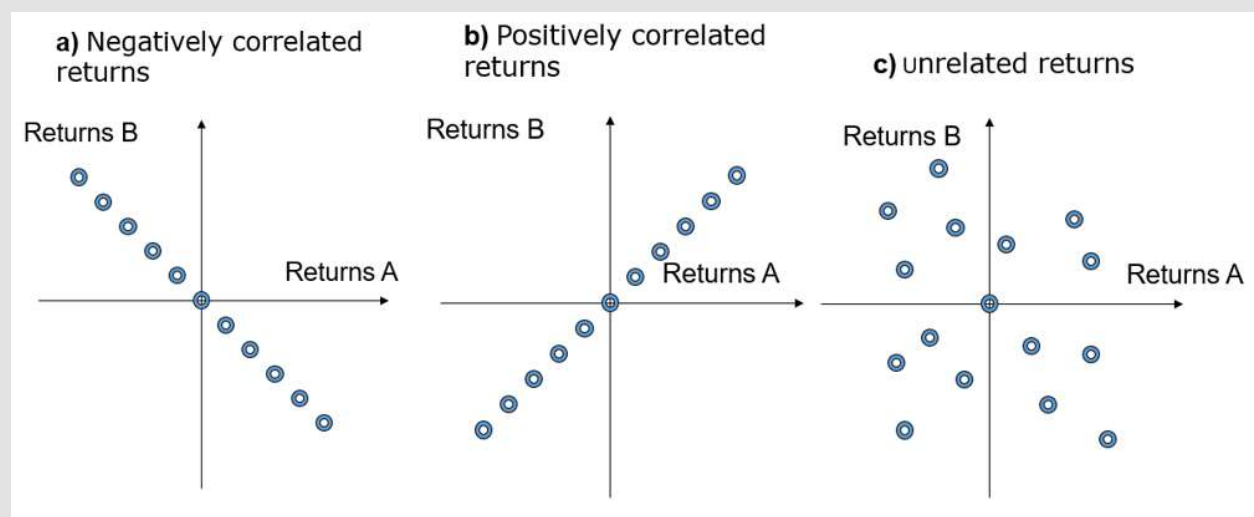
If you want to decrease the risk of your portfolio, you should select assets with correlation close to −1 (negatively correlated returns of individual assets). Let us use our example of Apple and Google to explain why and to calculate the risk of this portfolio.

For a two asset portfolio we use following formula to calculate the risk:

$$\sigma_P = \sqrt[2]{w_1^2 * \sigma_1^2 + w_2^2 * \sigma_2^2 + 2w_1 \, w_2 COR_{12}\sigma_1 \, \sigma_2}$$

In our case the portfolio risk of Apple (20%) and Google (80%) with correlation 0,0777 and previously calculated returns and risk is:

$$\sigma_P = \sqrt[2]{0{,}2^2 * 37{,}52^2 + 0{,}8^2 * 25{,}29^2 + 2 * 0{,}2 * 0{,}8 * 0{,}0777 * 37{,}52 * 25{,}29} = 22{,}15\%.$$

So basically, if you combine two assets with risk 37,52% and 25,29% in a ratio of 20% to 80% your risk would be only 22,17%. This is lower than individual risk of Apple and Google. Therefore, to decrease the risk a good portfolio is a good strategy. The reason is the correlation of the assets. If you combine assets which returns are negatively correlated, the losses generated by asset A will be covered by gains of asset B and therefore your investment is less risky.


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In our case the portfolio risk and return depends on the weights on Apple and Google in the portfolio. Because you can invest different amounts of money into Apple and Google. You can build a portfolio with Apple share 10%, 20%, 30% or even 99% and Google share would be 90%, 80%, 70%, or 1%. Anytime you change the weights the risk and return of your portfolio changes. Details and results are in the following table and figure

Effect of proportion of Apple on the risk and return of Apple and Google portfolio

Proportion of Apple in portfolio in %	Portfolio risk σp	Portfolio return R(p)
0	25.29%	22.56%
10	23.35%	23.92%
20	22.12%	25.28%
30	21.70%	26.64%
40	22.15%	28.00%
50	23.42%	29.36%
60	25.38%	30.72%
70	27.90%	32.08%
80	30.82%	33.44%
90	34.05%	34.80%
100	37.52%	36.16%

Effect of proportion of Apple on the risk and return of Apple and Google portfolio
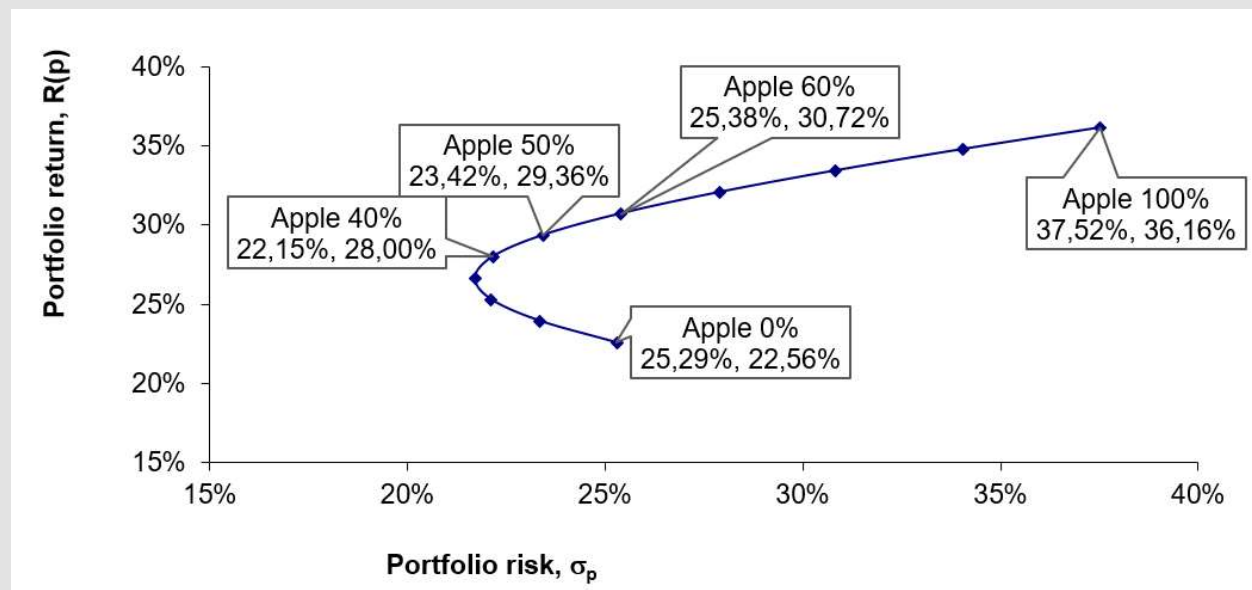
Based on the figure we can conclude that investor should consider a portfolio with 30% to 100% of Apple proportion. Proportion of Apple below 30% is not efficient, because we can earn higher return with the same level of risk with other proportions.

The more assets you have in your portfolio, the lower the risk. In real life the selection of assets in a portfolio is done by experts, which combine assets for you. You only decide on the level of total risk you want do face. Risk averse people prefer low risk and select conservative portfolios of bonds. Risk seekers prefer high return and select dynamic portfolios of stocks. Your overall portfolio consists of all your investments including house, flat, gold or your business and not only from financial investments.

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